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Experimental investigation of the motion trajectory of solid particles inside the hydrocyclone by a Lagrange method

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Abstract

Experimental investigations on the motion trajectory of solid particles inside the hydrocyclone have been successfully carried out by using a high-speed motion analyzer to track the particle movement. For each single particle, the motion trajectory is featured with stochastic characteristic; however, for the overall samples of particles, their motions hold the statistical property. The initial position of particles at the entrance of hydrocyclone heavily affects the motion trajectory of particles inside hydrocyclone and consequently the separation performance. An inlet with pre-sedimentation effect should be very helpful for the separation inside hydrocyclones. The results in this study are valuable for understanding the stochastic and statistical behaviors of particle motion in the separation process inside hydrocyclones, and provide some valuable information for finding some effective way to improve the separation performance in the hydrocyclone.

Keywords: Hydrocyclones; Separations; Particle processing; Fluid mechanics; Visualisation; Motion trajectory

1. Introduction

Hydrocyclones are getting more and more interest from various industries, because of their obvious advantages such as simple structure, large capacity, low cost and small volume. Beside a large amount of applications in mineral processing, hydrocyclone separation technique has gotten numerous applications recently in environmental engineering [1–8], petrochemical engineering [9–15], food engineering [16–19], electrochemical engineering [20,21], bioengineering [22–25], pulping process [26–28] and so on. More and more attentions are being paid to hydrocyclones.

Although the geometric structure of a hydrocyclone is simple, the motion of particles inside the hydrocyclone is very complicated. To understand the separation behavior of hydrocyclones and to obtain a theoretical foundation for structure optimization and efficiency improvement of hydrocyclones, it is essential to make the motion of particles in the hydrocyclones as clear as possible. Therefore many investigations have been made on the motion of solid particles in hydrocyclones previously. Almost all previous experimental works on the motion of solid particles in hydrocyclones have made by adopting an Euler method, *i.e.*, all the works have been focused on the flow field by studying the motion of particle passing through some fixed positions in hydrocyclones [29–31]. However, to understand the stochastic and statistical behaviors of particle motion in the separation process inside hydrocyclones, which have been considered as the key point to understand the separation behavior in hydrocyclones, it is very important and essential to investigate the motion of particles in the hydrocyclone by using a Lagrange method, *i.e.*, studying the motion of particles by tracking the particles in the hydrocyclone.

The objective of this study is to understand the stochastic and statistical behaviors of the motion of solid particles in the hydrocyclone by adopting a Lagrange method. The investigation was experimentally carried out by using a high-speed motion analyzer (HSMA) system to track the motion trajectory of solid particles inside hydrocyclones. The stochastic and statistical characteristics of the motion of solid particles in the hydrocyclone were analyzed according to the measured motion trajectory of solid particles. The experimental results in this study are valuable for understanding the separation process in the hydrocyclone.

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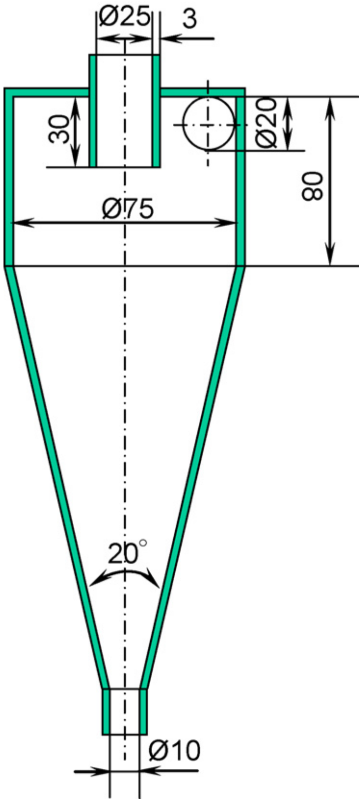

Fig. 1. Geometry of the hydrocyclone.

2. Experimental

2.1. Apparatus and instrument

The hydrocyclone was made of transparent Perspex and polished. The geometric parameters of the hydrocyclone were designed according to Rietema's optimum geometry for separation [32], as illustrated in Fig. 1. The experimental instrument and apparatus system were arranged as shown in Fig. 2.

A high-speed motion analyzer (HSMA) system was used to track the motion trajectory of solid particles inside the hydrocyclone. It is composed of a high-speed camera, a data processor, a video recorder, a monitor and a computer. When solid particles were fed into the hydrocyclone, a movie of the solid motion was taken by the high-speed camera. Then, the movie was transformed into frames. The motion trajectory of solid particles in the hydrocyclone was obtained by plotting the position of particles at different times, as shown in Fig. 3.

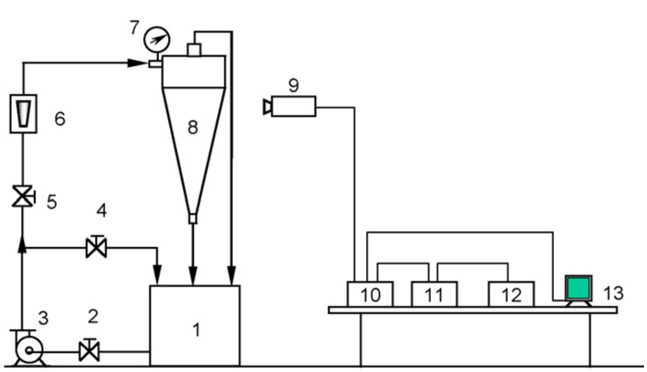

Fig. 2. Schematic illustration of the test system: (1) tank; (2) valve; (3) pump; (4) valve; (5) valve; (6) flowmeter; (7) pressure gauge; (8) hydrocyclone; (9) camera; (10) data processor; (11) video; (12) monitor; (13) computer.

2.2. Particles and injection positions

The liquid phase was water. To be captured by the fast camera, the solid particles should be easy to be found in the water, *i.e.*, the color should be dark and the size should be large enough to be seen in the movie, but the density should not be too large to study the separation behaviors. In the experiments, we chose a kind of vegetable seeds with black surface and with density of 1.14 g/cm^3 as the solid particles. The particle size was graded carefully before experiments, and particles with uniform size were selected for a batch of observations. The inlet flow rate and pressure of the feed was adjusted by the valves and both were maintained constantly throughout all the experiments.

To study the effect of the initial position of particles at the inlet of hydrocyclone on the particle motion inside the hydrocyclone, we injected the particles into hydrocyclone from three different positions of the inlet section, as shown in Fig. 4. The three positions were in the same cross-section, in which injection point 1 was near the outer side of the inlet wall, point 2 was in the central axis, and point 3 was near inner side of the inlet wall.

3. Results and discussion

3.1. Effect of particle size on the motion trajectory of solid particles

With injecting particles from the fixed position point 1, particles with diameters of 700, 800 and 900 μm were used to study the motion trajectories inside the hydrocyclone. As listed in Table 1, when the particle diameter was 700 μm, most particles (72%) were discharged from the vortex finder; on the other hand, when the particle diameter was 900 μm, most particles (71%) were separated and went to the underflow orifice. The larger the particle diameter, the larger the centrifugal force on the particles, as a result the more particles were separated into underflow. The smaller the particle diameter, the more particles entered the overflow without entering the cone section, *i.e.*, the harder for the particles to be separated into underflow.

Figs. 5 and 6 show the typical motion trajectories of particles with different diameters that discharging from underflow and overflow, respectively.

In Fig. 5, when the particle diameter was 700 μm, the particle rotated and wandered in the middle area of cylinder and cone body. The reason might be that the particle fell into the locus of zero vertical velocity (LZVV). On the other hand, such phenomena were not observed for the particles with diameters of 800 and 900 μm. Table 2 listed the statistical residence time of particles with different diameters that finally discharging from underflow. With increasing the diameter, both the average residence time and the shortest residence time of the particle inside

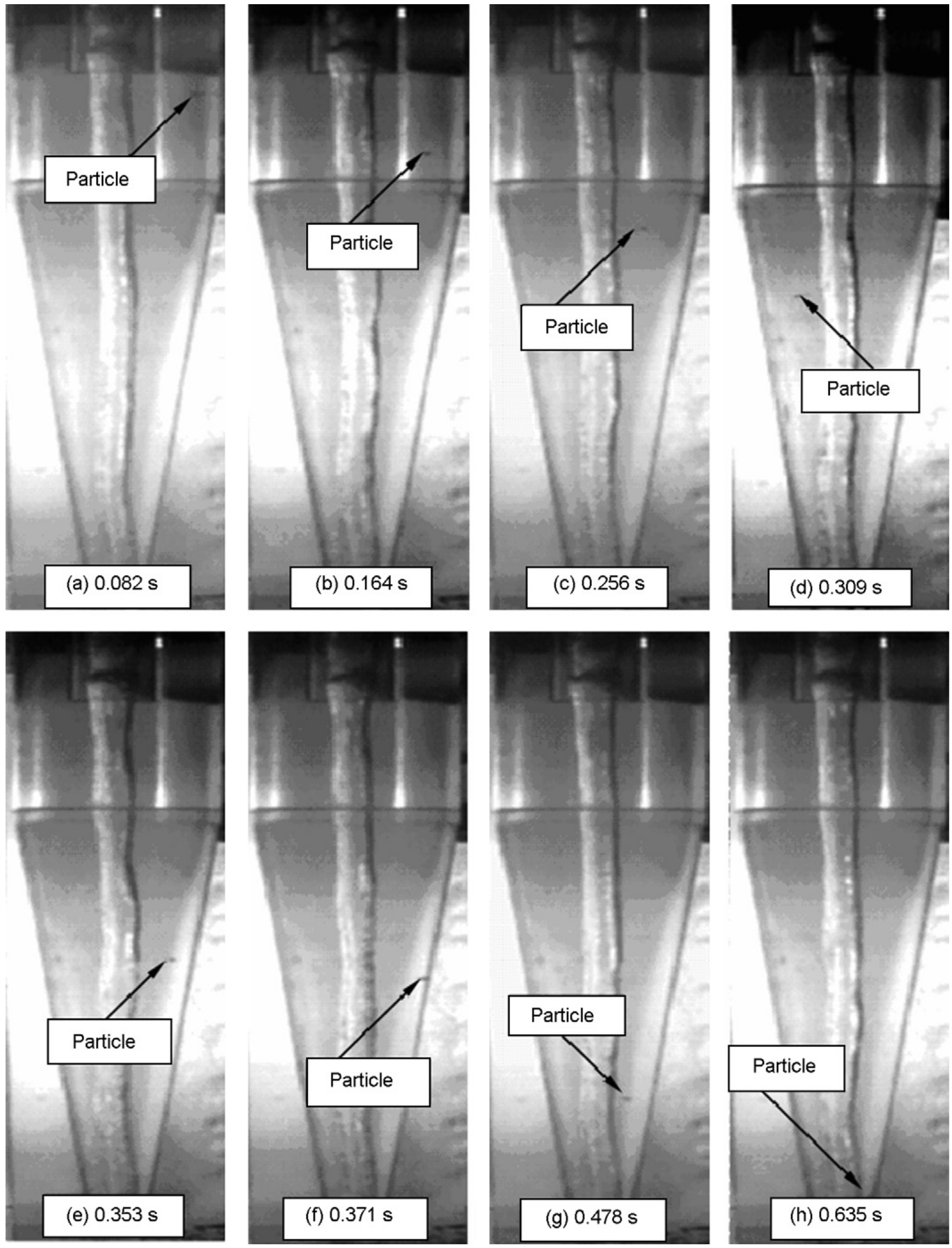

Fig. 3. Photographs of particle motion inside the hydrocyclone.

Table 1
Statistical percentage of particles with different diameters entering underflow and overflow (particles were injected into the hydrocyclone from point 1)

Particle diameter (μm)	Percentage entering underflow	Percentage entering overflow		
		Total	Entering into the cone section first	Never entering into the cone section
700	28.0	72.0	36.0 (50.0%*)	36.0 (50.0%)
800	55.0	45.0	27.0 (60.0%)	18.0 (40.0%)
900	71.0	29.0	18.0 (62.0%)	11.0 (38.0%)

Note: *the percentage in the parentheses is that calculated by taking the overflow as 100%.

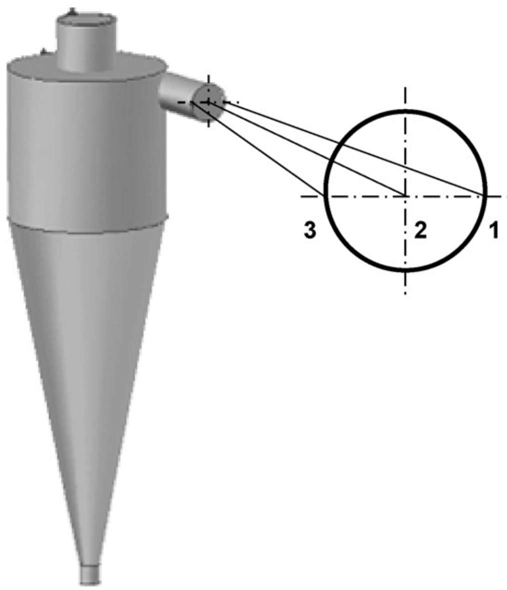

Fig. 4. Location points for injecting particles into the hydrocyclone: (1) outer position; (2) central position; (3) inner position.

Table 2
Statistical residence time of particles with different diameters that finally discharging from underflow (particles were injected into the hydrocyclone from point 1)

Particle diameter (μm)	Residence time of particles inside the hydrocyclone (s)		
	Longest time	Shortest time	Average time
700	1.467	0.620	1.060
800	3.212	0.541	0.831
900	1.099	0.498	0.712

Table 3
Statistical residence time of particles with different diameters that finally discharging from overflow (particles were injected into the hydrocyclone from point 1)

Particle diameter (μm)	Residence time of particles inside the hydrocyclone (s)		
	Longest time	Shortest time	Average time
700	0.722	0.207	0.346
800	1.817	0.400	0.750
900	1.517	0.543	0.856

the hydrocyclone decreased. That is, with increasing the particle diameter, the separation process finished faster. However, a particle with diameter of 800 μm held the longest residence time. Fig. 7 shows the motion trajectory and 2D motion path curve of the particle that discharging from the underflow with the longest residence time inside the hydrocyclone. The particle moved several circles between the inner helical flow and outer helical flow. The reason might be that the particle fell into the locus of zero vertical velocity, and at the same time the particle suffered from a radial velocity fluctuation because of the turbulence inside the hydrocyclone. Therefore, the particle spent a lot of time to get the way to the underflow.

In Fig. 6, when the particles finally discharged from the overflow, the bigger the diameter of the particle, usually the longer the axial distance that the particles traveled. When the particle had larger diameter, larger centrifugal force was acted on it, therefore the inertia of its motion should be larger, as a result it could arrive a longer axial distance from the entrance. Table 3 shows the statistical residence time of particles with different diameters that finally discharging from overflow. With increasing the particle diameter, both the average residence time and the shortest residence time became longer. That is, the larger the particles, the more difficult for them to enter the vortex finder.

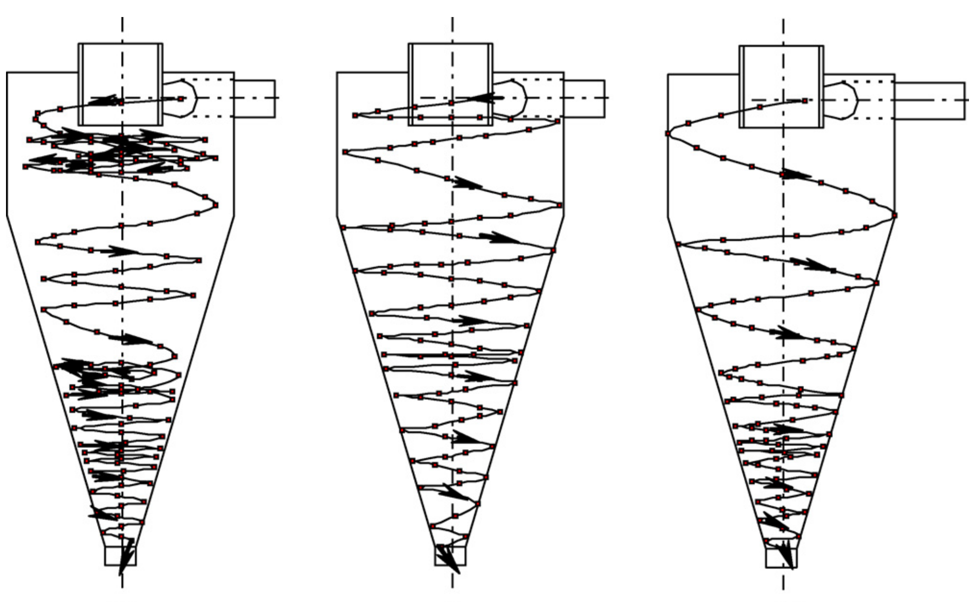

Fig. 5. Typical motion trajectories of particles with different diameters that discharging from underflow: (a) particle diameter = 700 μm; (b) diameter = 800 μm; (c) diameter = 900 μm. The particles were injected from point 1.

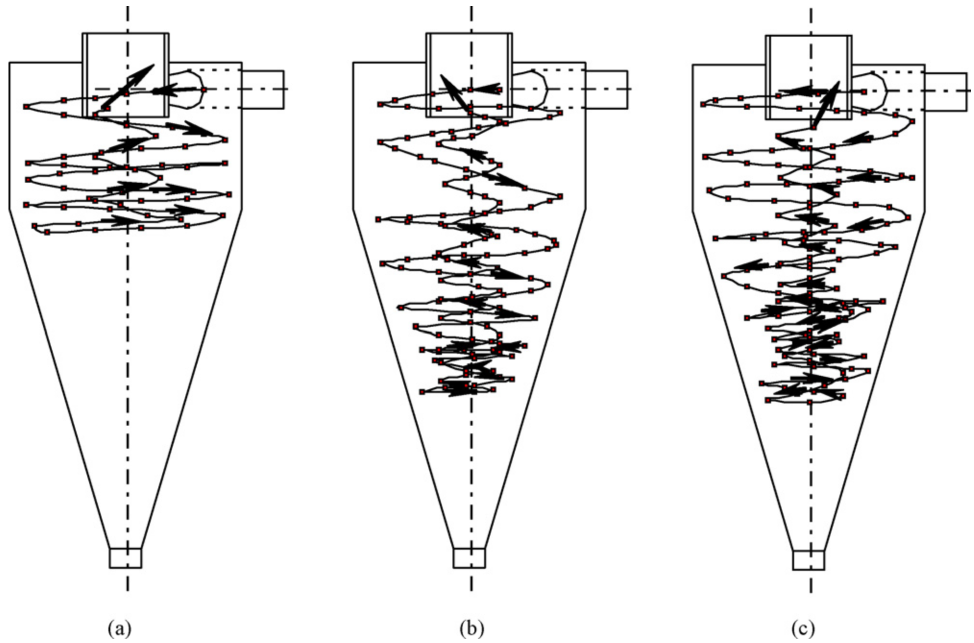

Fig. 6. Typical motion trajectories of particles with different diameters that discharging from overflow: (a) particle diameter = 700 μm; (b) diameter = 800 μm; (c) diameter = 900 μm. The particles were injected from point 1.

3.2. *Effect of the injection position on the motion trajectory of solid particles*

To study the effect of the injection position on the motion trajectory of solid particles inside the hydrocyclone, the particle size was fixed as 800 μm.

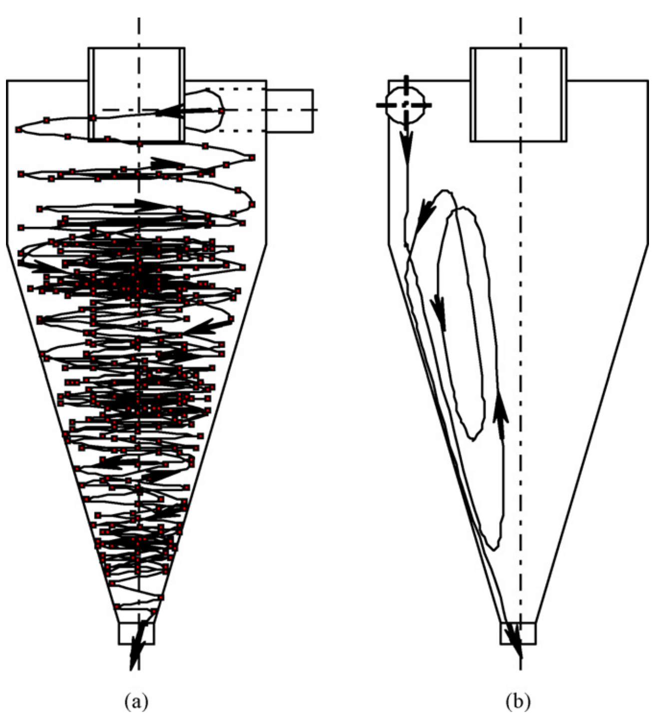

Fig. 7. (a) The motion trajectory and (b) 2D motion path curve of the particle that discharging from the underflow with the longest residence time inside the hydrocyclone.

When the particles were injected into the hydrocyclone from different positions, some of them were discharged out from the underflow orifice, while some were discharged from the vortex finder. The statistical percentages of particles entering underflow and overflow when they were injected into the hydrocyclone at different positions are listed in Table 4. Most particles were separated from the underflow when the particles were injected into the hydrocyclone from the position point 1, while least particles were separated from the underflow when they were injected from the position point 3, and those particles injected from the position point 2 were in the medium condition. When the particles were injected into the hydrocyclone from the point 1, only 4.5% of them discharged from overflow with never entering the cone section; on the other hand, when the particles were injected from the position point 3, 35.5% of them discharged from overflow with never entering the cone section. That is to say, the initial position of the particles at the entrance section of the hydrocyclone is very important for them to get separated or not.

Fig. 8 shows the typical motion trajectories of particles inside the hydrocyclone that discharging from underflow, Fig. 9 shows that entering the cone section first and then discharging from overflow, and Fig. 10 discharging from overflow and never entering the cone section, respectively.

As shown in Fig. 8, for those particles discharging from underflow, when the particle was injected into the hydrocyclone from the point 1, it arrived to the hydrocyclone wall very soon. When the particle was injected from the point 2, it arrived to the hydrocyclone wall near the cylinder–cone intersectant area. Whereas, when the particle was injected from point 3, it arrived to the hydrocyclone wall in the middle part of cone body. The reason was that the radial distance for the particles to arrive the hydrocyclone wall was different when they were injected into hydrocyclone from different position. Furthermore, Fig. 8 shows

Table 4
Statistical percentage of particles entering underflow and overflow when they were injected into the hydrocyclone at different positions (particle diameter = 800 μm)

Injection point	Percentage entering underflow	Percentage entering overflow		
		Total	Entering into the cone section first	Never entering into the cone section
1	55.0	45.0	40.5 (90.0%*)	4.5 (10.0%)
2	50.0	50.0	38.0 (76.0%)	12.0 (24.0%)
3	26.0	74.0	38.5 (52.0%)	35.5 (48.0%)

Note: *the percentage in the parentheses is that calculated by taking the overflow as 100%.

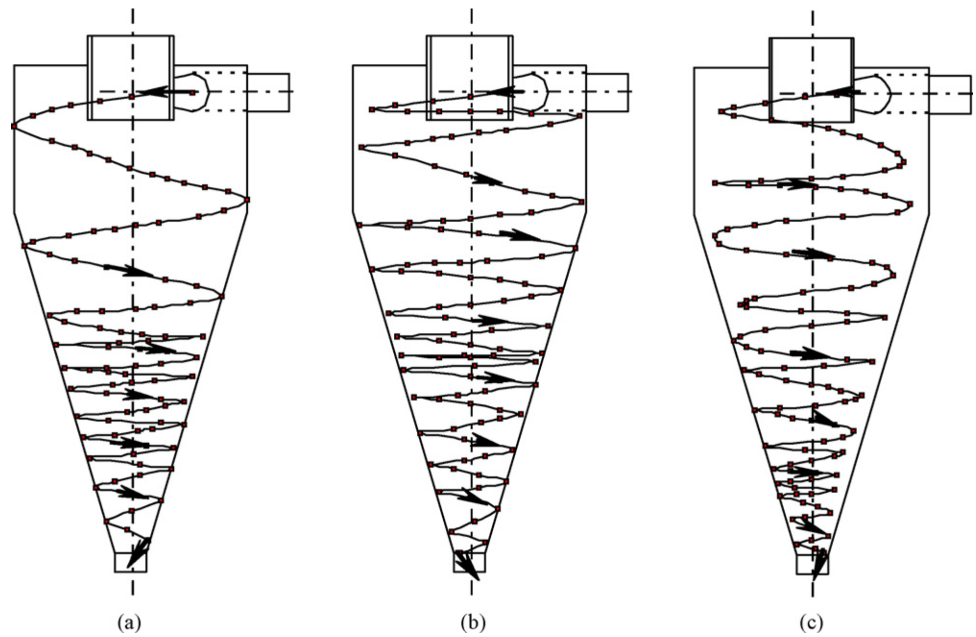

Fig. 8. Typical motion trajectories of particles inside the hydrocyclone that discharging from underflow: (a) particles injected from point 1; (b) injected from point 2; (c) injected from point 3.

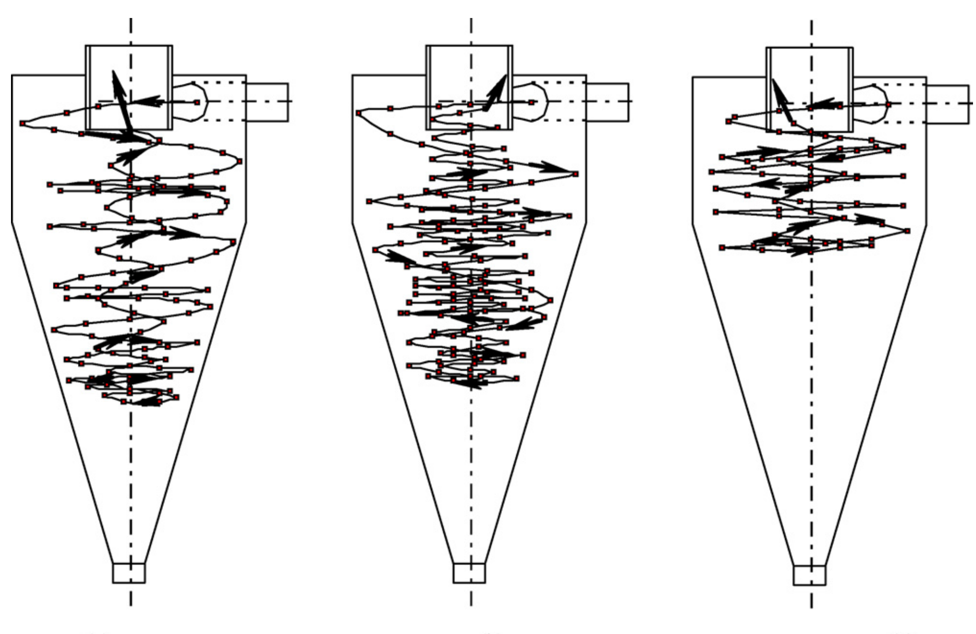

Fig. 9. Typical motion trajectories of particles inside the hydrocyclone that entering the cone section first and then discharging from overflow: (a) particles injected from point 1; (b) injected from point 2; (c) injected from point 3.

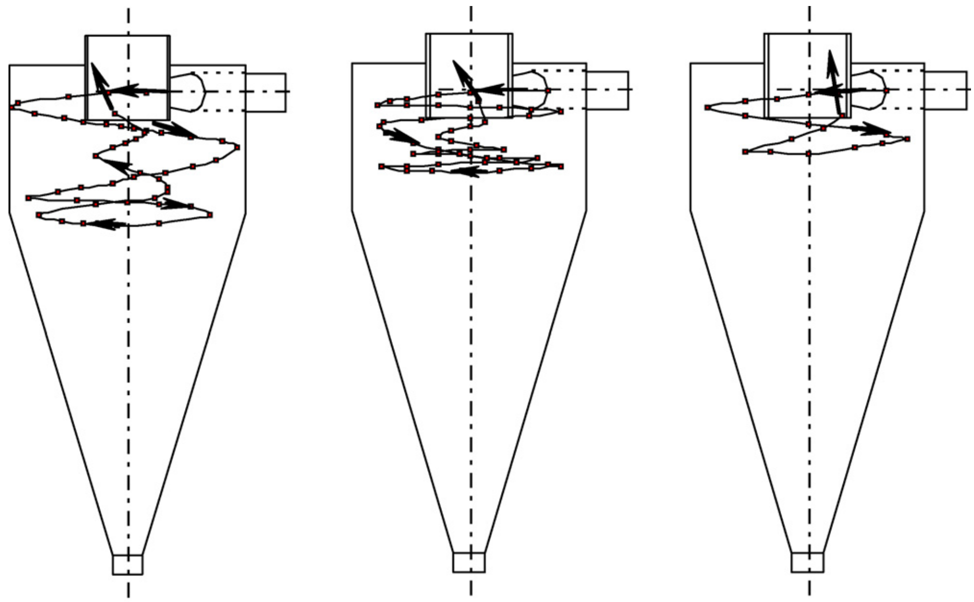

Fig. 10. Typical motion trajectories of particles inside the hydrocyclone that discharging from overflow and never entering the cone section: (a) particles injected from point 1; (b) injected from point 2; (c) injected from point 3.

that the screw pitch of the motion trajectory existed differences in the cylinder body. The screw pitch of the motion trajectory of particles injected from point 1 was larger in the cylinder body than those injected from point 2 and point 3. When the particles entered into the cone body, the screw pitches gradually became small. According to the typical tangential velocity profile $u_\theta r^n = c$ inside the hydrocyclone [29,30], with increasing the radial position r, the tangential velocity u_θ would decrease, and then the particle would spend longer time to rotate a cycle; as a result, the axial screw pitches were different.

Table 5 shows the statistical residence time of particles inside the hydrocyclone that injected into cyclone at different positions and finally discharging from underflow. The results showed that, when the particles were injected into the hydrocyclone from the position point 2, both the longest residence time (3.212 s) and the average residence time (1.088 s) were the longest. The reason might be that they were easy to reach the critical position for separation, *i.e.*, they were wanderers near the critical position for reaching the main flow of underflow. On the other hand, when the particles were injected into the hydrocyclone from position point 1, the residence time inside the hydrocyclone was shortest.

Table 5
Statistical residence time of particles inside the hydrocyclone that injected into cyclone at different positions and finally discharging from underflow (particle diameter = 800 μm)

Injection point	Residence time of particles inside the hydrocyclone (s)		
	Longest time	Shortest time	Average time
1	1.354	0.441	0.825
2	3.212	0.447	1.088
3	1.849	0.628	0.828

Fig. 9 shows typical motion trajectories of particles inside the hydrocyclone that entering the cone section first and then discharging from overflow. When those particles were injected from position point 1 and finally discharged from overflow, they usually could arrive at the axial position of about two-third of the cone height from cylinder–cone intersectant plane. On the other hand, when the particles were injected into the hydrocyclone from position 2, most particles could only arrive at the middle part of the cone body. When the particles were injected from position point 3, most particles could only arrive at the adjacent area of cylinder–cone intersectant plane. Because the initial radial positions were different when the particles were injected into hydrocyclone from the mentioned three position points, the centrifugal force acted on the particles injected from point 1 was the largest, while that on the particles injected from point 3 was the smallest. From Fig. 9 we can also find that the screw pitch of the motion trajectory existed differences between those in inner and outer helical flow. The screw pitch of the motion trajectory of particles in inner helical flow seemed to be more regular than that in outer helical flow.

Fig. 10 shows the typical motion trajectories of particles inside the hydrocyclone that discharging from overflow and never entering the cone section. Because of the difference in the centrifugal force at initial injection point as mentioned above, the particles injected into the hydrocyclone from position point 3 most easily reached the vortex finder, while it was the most difficult for those from position point 1 to get into the vortex finder.

Table 6 shows the statistical residence time of particles inside the hydrocyclone that injected into hydrocyclone at different positions and finally discharging from overflow. The results showed that the average residence time of particles injected from point 1 was the longest, and that of particles injected from point

Table 6
Statistical residence time of particles inside the hydrocyclone that injected into cyclone at different positions and finally discharging from overflow (particle diameter = 800 μm)

Injection point	Residence time of particles inside the hydrocyclone (s)		
	Longest time	Shortest time	Average time
1	0.928	0.722	0.838
2	1.898	0.212	0.601
3	0.493	0.222	0.440

3 was the shortest one, just as expected. On the other hand, for arbitrary single particle, the residence time of particles injected from position point 2 was the longest. The motion trajectory and 2D motion path curve of the particle that discharging from the vortex finder with the longest residence time inside the hydrocyclone are shown in Fig. 11. The particle might happen to fall into the locus of zero vertical velocity, which led to the particle just rotating with little axial moving, and then the residence time in the hydrocyclone became the longest one.

The experimental results showed that, even though the particle size and initial position at the entrance of hydrocyclone are the same, the motion trajectories of particles inside the hydrocyclone are still different and random. The point is different from some previous hypothesis and presumes used in the numerical simulation. Although the motion trajectory of every particle is random, and the chance for a single particle being separated into the underflow is stochastic; for the whole involved particles, they hold statistical characteristics for their motions. Statistical analysis of our experiment results (listed in Table 4) showed that the percentage of particles of injected from point 1 to be separated into underflow was much more than that injected from

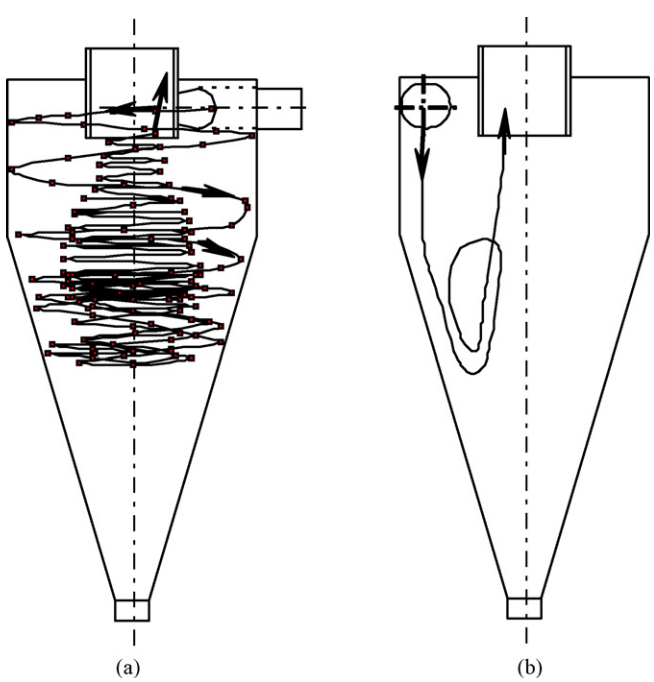

Fig. 11. (a) The motion trajectory and (b) 2D motion path curve of the particle that discharging from the vortex finder with the longest residence time inside the hydrocyclone.

position point 3. That is, the separation efficiency of particles in the hydrocyclone was heavily influenced by the initial position of the particle at the entrance of hydrocyclone. According to the results, design of inlet pipe with pre-sedimentation effects should be very helpful for the separation inside hydrocyclones. For example, inlet pipe with involute or helical shape should be better than that with common tangential shape for the separation process inside the hydrocyclone, because there exists centrifugal sedimentation in the inlet pipe with involute or helical shape due to the function of centrifugal force, but no centrifugal sedimentation exists in the inlet pipe with traditional tangential shape because the tangential inlet pipe is straight and no centrifugal force acts on the particles in the inlet pipe.

4. Conclusions

Experimental investigations on the motion trajectory of solid particles inside the hydrocyclone have been successfully carried out by using a high-speed motion analyzer. For each single particle, the motion trajectory is featured with stochastic characteristic; however, for the overall samples of particles, their motions hold the statistical property. The initial position of particles at the entrance of hydrocyclone heavily affects the motion trajectory of particles inside hydrocyclone and consequently the separation performance. An inlet with pre-sedimentation effect should be very helpful for the separation inside hydrocyclones. The results in this study are valuable for understanding the stochastic and statistical behaviors of particle motion in the separation process inside hydrocyclones, and from which we can find some effective way to improve the separation performance in the hydrocyclone, e.g., designing the inlet pipe with pre-sedimentation effects could be very helpful for the separation inside hydrocyclones.

Acknowledgements

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